A New Solid Subdivision Based on Box Splines

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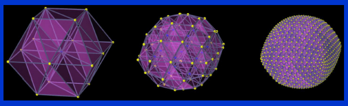
Abstract



- We propose a novel solid subdivision scheme based on tri-variate box splines over tetrahedral tessellations in 3D. The subdivision hierarchy can be easily constructed by calculating new vertex, edge, and cell points at each level as affine combinations of neighboring control points at the previous level. Obtained from tri-variate box splines, our subdivision solid ensures high-order continuity. To further demonstrate the modeling potential, we conduct several solid modeling experiments including free-form deformation. We hope to demonstrate that our box-spline solid advances the current state-of-the-art in solid modeling in the following aspects: (1) unifying CSG, B-rep, and cell decomposition within a popular subdivision framework; (2) overcoming the shortfalls of tensor-product spline models; (3) generalizing both subdivision surfaces and free-form spline surfaces to a solid representation of arbitrary topology; and (4) taking advantage of triangle-driven, accelerated graphics hardware.

1. Introduction

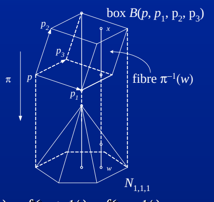
- **Solid Representations**
  - *Implicit functions: CSG models, blobby models, algebraic solids, etc.*
  - *Parametric representations: Bernstein-Bezier solids, B-spline solids, tensor-product based solids*
  - *Cell decomposition technique: Voxel spaces, octree, etc.*
- **Benefits of Subdivision**
  - *Unified representation*
  - *Numerically efficient way to evaluate*
  - *Can handle arbitrary topological domain*
  - *Inherently supports multiresolution / LOD*

► *Combining a subdivision with a solid representation could give us huge benefit.*

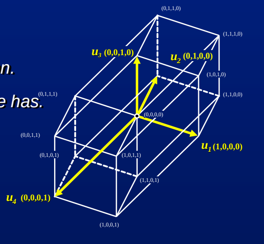
- **Previous Works on Multivariate Splines**
  - *Bernstein-Bezier volume (Lasser et al., CAGD '85)*
  - *Trivariate B-spline solid (Greissmair et al., Eurographics '89)*
  - *Tensor-product solid (MacCracken et al., SIGGRAPH '96)*
  - *Prism spline filter (McCool, SIGGRAPH '95)*
  - *Box spline filter (Peters, SM '97)*

► *Most works are related to 3D mesh for free-form deformation and filtering, not modeling purposes.*

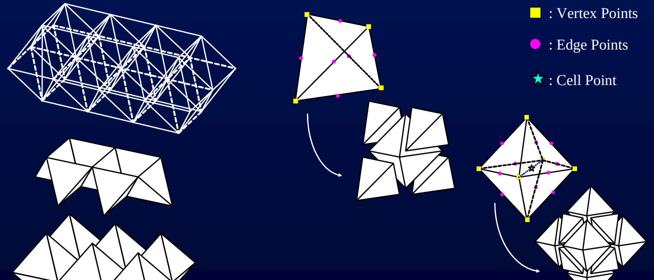
2. Trivariate Box Spline Volumes



- **Univariate B-Splines**
  - *Defined by convolution and recursion;* $B_1 = \chi_{(-1/2,\, 1/2)}$, $B_n = B_{n-1} * B_1$
  - *Integer shifts form a partition of unity;* $\Sigma_j B_n(x - j) = 1$
  - *Truncated powers;* $B_n(x) = 1/(n-1)! * \Delta^n x_+^{n-1}$ *where* $\Delta f(x) = f(x + 1/2) - f(x - 1/2)$, $x_+ = \max(x, 0)$
- **Box Spline Solids**
  - *Defined by projection of hypercube (defined by* direction sets*) into lower dimension.*
  - *Satisfies the properties that univariate B-spline has.*
  - *Projection direction is critical.*



- **3D Regular Mesh and the Masks**
  - *Projection of direction sets (2 for each* u_i*)*
  - *It does not form a tessellation of 3D space.*
  - *We use* **Octet-truss** *structure, which comprises of octahedra with tetrahedra in between. → Regular valance for each vertex / edges*



Octet-truss and
Its Bisection via Edge Splitting

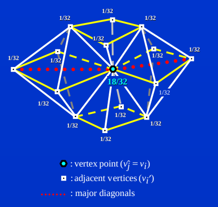
3. Solid Subdivision Rules

- **Subdivision** → Decompose a box spline into an affine combination of box splines with smaller supports.

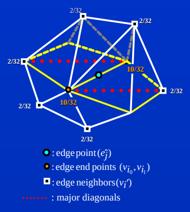
$$N_i = \frac{1}{2}\sum_{\hat{j}} \alpha_{i,\hat{j}} \hat{N}_{\hat{j}} \quad \text{where } a_{i,\hat{j}} = \begin{cases} 2 \text{ if } i = \hat{j} \\ 1 \text{ if } i \text{ is adjacent to } \hat{j} \\ 0 \text{ otherwise} \end{cases}$$

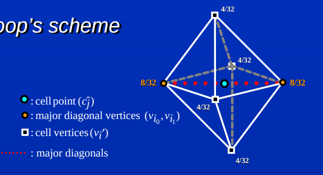
$$N_i = \frac{1}{2}\sum_{\hat{j}} \alpha_{i,\hat{j}} \left(\frac{1}{c} \sum_{\hat{k}} \alpha_{\hat{j},\hat{k}} N_{\hat{k}} \right)$$

$$S = \sum_i v_i N_i = \sum_{\hat{j}} w_{\hat{j}} N_{\hat{j}} \quad \text{where } w_j = \frac{1}{2c} \sum_i \sum_{\hat{k}} \alpha_{i,\hat{j}} \alpha_{\hat{j},\hat{k}} v_i$$



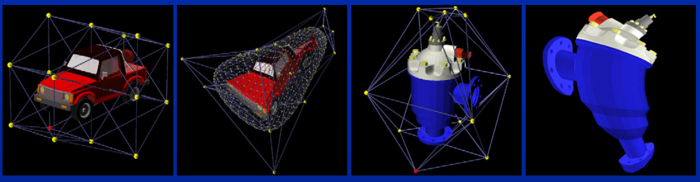
- **Rules**
  - *Vertex Points* $v_{\hat{j}} = \frac{18}{32} v_i + \frac{1}{32} \sum_{i'} v_{i'}$
  - *Edge Points* $e_{\hat{j}} = \frac{10}{32} (v_{i_0} + v_{i_1}) + \frac{2}{32} \sum_{i'} v_{i'}$
  - *Cell Points* $c_{\hat{j}} = \frac{8}{32} (v_{i_0} + v_{i_1}) + \frac{4}{32} \sum_{i'} v_{i'}$
  - *Boundary; follows modified Loop's scheme*



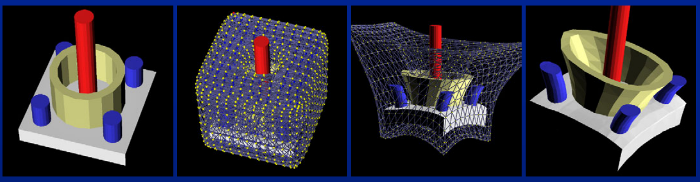


4. Applications

1. Free-form Deformation

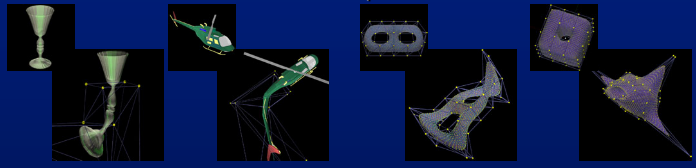


Free-form deformation is done by embedding an object into our subdivision solid and calculating corresponding barycentric coordinates.

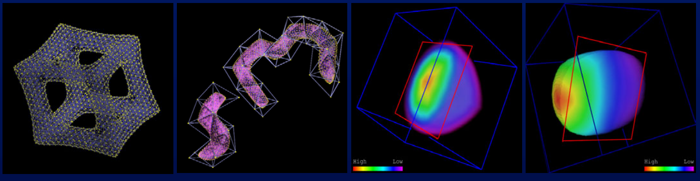


The control solid has non-trivial topology (with 1 hole). Subdivision process can treat it with ease.

2. Local Control and Direct Solid Manipulation



3. Interactive Modeling and Material Property Presentation



5. Conclusion and Future Works

- Superior flexibility due to tetrahedral based structure
- 12 DOF rather than 6 DOF of tensor product grid → Suitable for physics simulation
- High order of continuity with low degree of basis functions
- Fast and stable subdivision evaluation of solid itself
- Arbitrary topology can be handled easily
- Further study is required for extraordinary analysis and boundary representation