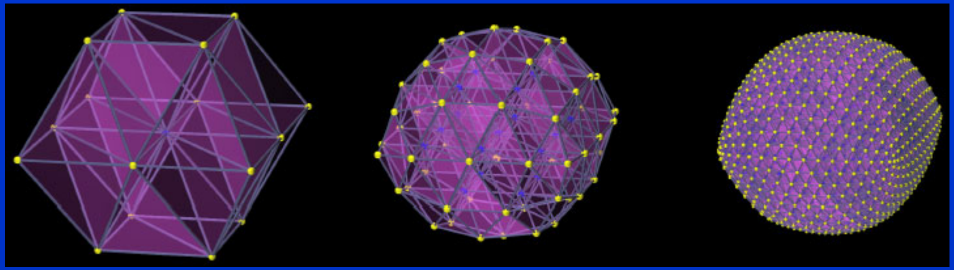

A New Solid Subdivision Scheme based on Box Splines

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Overview

1. Introduction
2. Multivariate Splines
3. Trivariate Box Splines
4. Solid Subdivision Rules
5. Applications
6. Conclusion and Future Works

1. Introduction [1]

Solid Modeling

- Desired in many engineering and manufacturing applications

Solid Representations

- Implicit functions: CSG models, blobby models, algebraic solids, etc.
- Parametric representations: Bernstein-Bezier solids, B-spline solids, tensor-product based solids.
- Cell decomposition technique: Voxel spaces, octree, etc.

1. Introduction (2)

Implicit Solids

$$w = f(x, y, z), \; w = w_0$$

- Easy to define interior, exterior and boundaries.
- Hard to evaluate, visualize and manipulate.

Parametric Solids

$$S(x, y, z) = \sum_i v_i N_i(x, y, z),$$

- Efficient evaluation.
- Localized control (spline-based).
- Tensor-product based.

1. Introduction (3)

Why Subdivision?

- Unified representation.
- Numerically efficient way to evaluate.
- Can handle arbitrary topological domain.
- Inherently supports multiresolution / LOD.
- Relatively simple in implementation (recursion).

► Combining a subdivision with a solid representation could give us huge benefit.

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1. *Introduction*
2. *Multivariate Splines*
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2. Multivariate Splines [(1)]

Previous Works on Multivariate Splines

- Bernstein-Bezier volume (Lasser et al., CAGD ’85).
- Trivariate B-spline solid (Greissmair et al., Eurographics ’89).
- Tensor-product solid (MacCracken et al., SIGGRAPH ’96).
- Prism spline filter (McCool, SIGGRAPH ’95).
- Box spline filter (Peters, SM ’97).

► Most works are related to 3D mesh for free-form deformation and filtering, not modeling purposes.

2. Multivariate Splines (2)

Univariate B-Splines

- Defined by convolution and recursion.

  $B_1 = \chi_{(-½, ½)}$, $B_n = B_{n-1} * B_1$

- Integer shifts form a partition of unity.

  $\Sigma_j B_n(x - j) = 1$

- Truncated powers.

  $B_n(x) = 1/(n-1)! * \Delta^n x_+^{n-1}$

  where $\Delta f(x) = f(x + ½) – f(x – ½)$, $x_+ = \max(x, 0)$

2. Multivariate Splines (3)

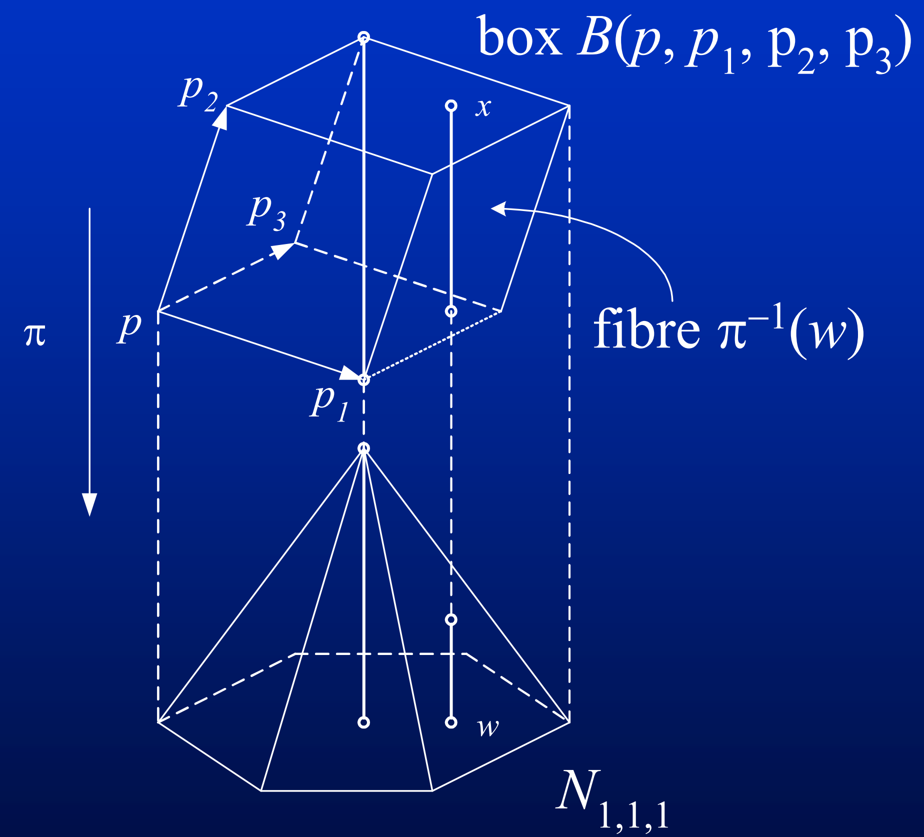
Extension to 3D – Tensor product

- $N(x, y, z) = B_n(x) \otimes B_n(y) \otimes B_n(z)$ where B_n; univariate
- Easy to understand / evaluate – basically every trick on univariate case will work.
- Requires high degrees for less continuity due to tensor product.
- Domain restriction – hexahedral lattice structures, 6 DOF on each point.

2. Multivariate Splines (4)

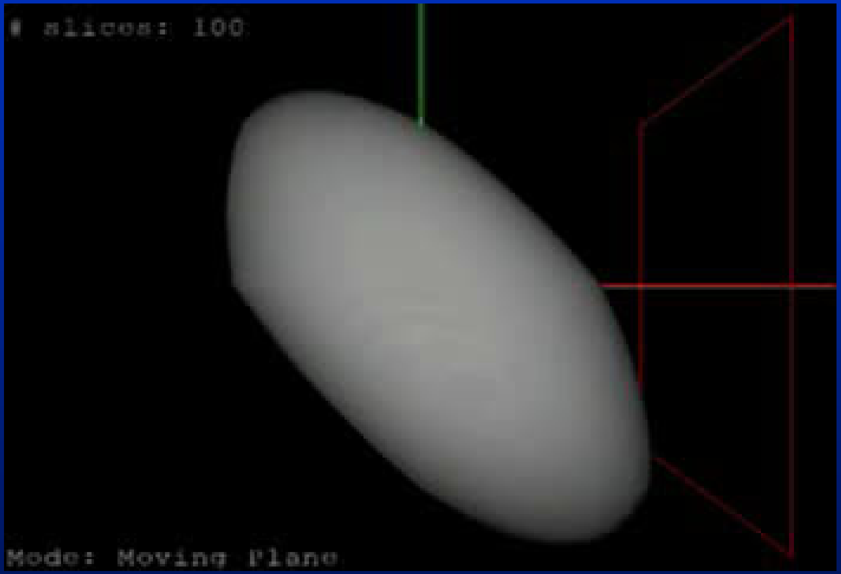
Box Spline Solid

- Defined by projection of hypercube (defined by *direction sets*) into lower dimension.
- Satisfies many properties that B-spline has.
  - *Recursive definition*
  - *Partition of unity*
  - *Truncated power*
- Projection direction is critical.

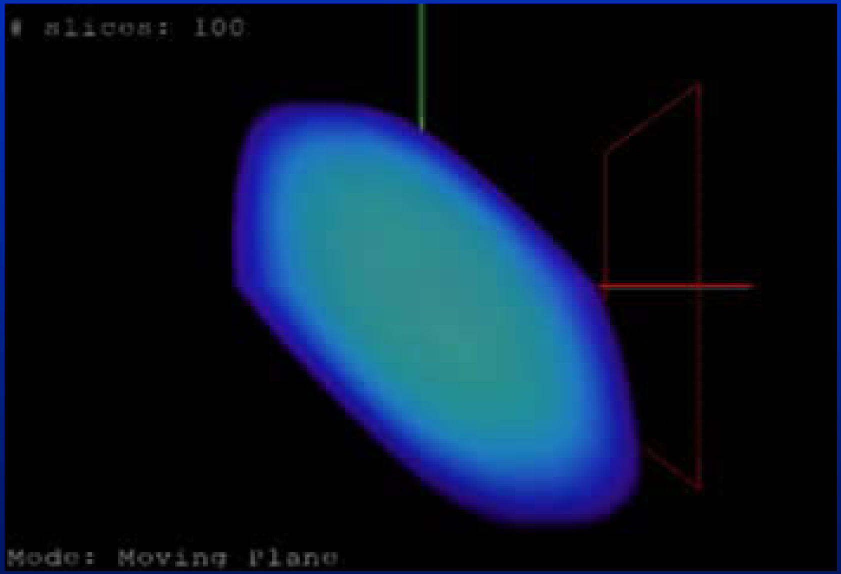


2. Multivariate Splines (5)

Basis Functions of Box Spline Solid

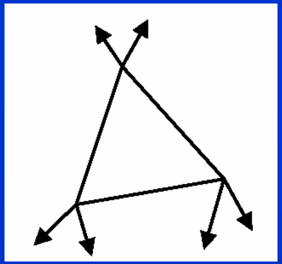


Basis (Gray Scale)



Basis (Spectral)

2. Multivariate Splines (6)



Trivariate Simplex Spline

- Similar definition to box spline, but instead of using hypercube, we are considering projections of simplices in higher dimension.
- The projection of a simplex is a complex in lower dimension. → Knot insertion problem.
- Also satisfies the properties.

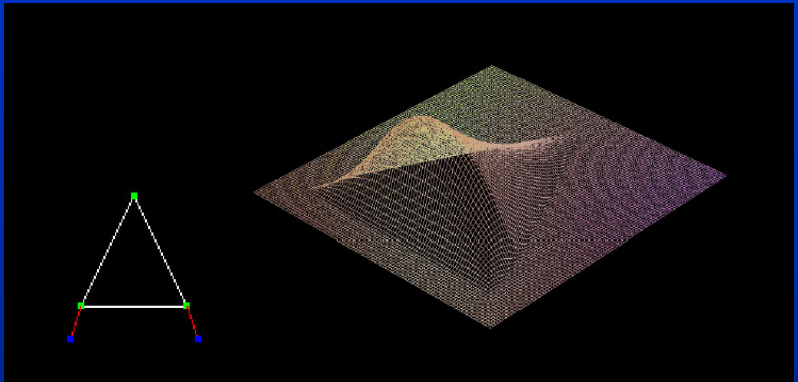


FIGURE 4.1 KNOT CONFIGURATION 1 OVER N_{110}

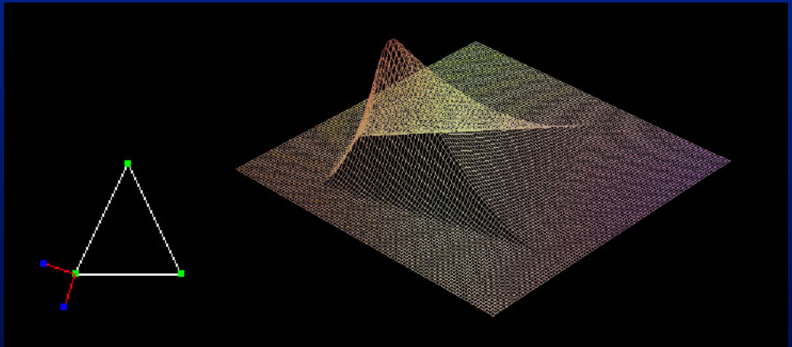


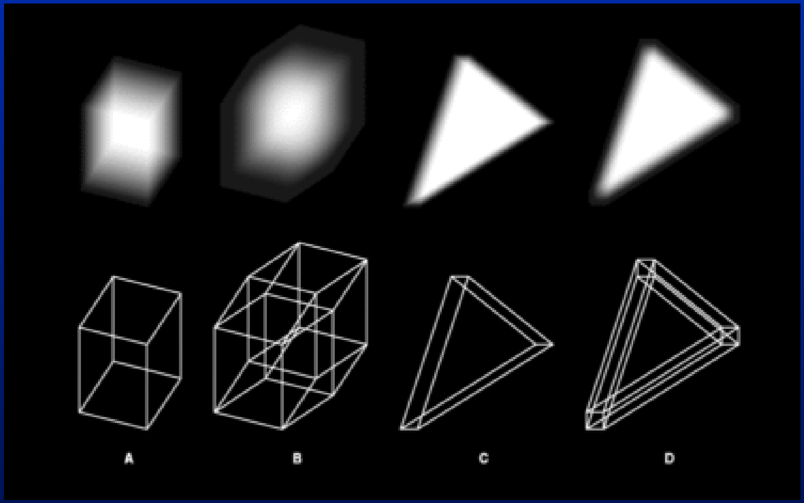
FIGURE 4.4 KNOT CONFIGURATION 4 OVER N_{200}

2. Multivariate Splines [7]

Generalized Idea; Polyhedral Splines

- B is a convex polyhedron or a parallelpiped (i.e. a hyperprism) in high dimension. U is a normalizer (unit volume).
- Properties: Convolution, recurrence and partition of unity

$$M_B(\boldsymbol{w}) = \frac{vol\big(\pi^{-1}(\boldsymbol{w}) \cap B\big)}{vol\, U(\boldsymbol{w})},$$



Courtesy of Michael D. McCool

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3. Trivariate Box Spline Solids (1)

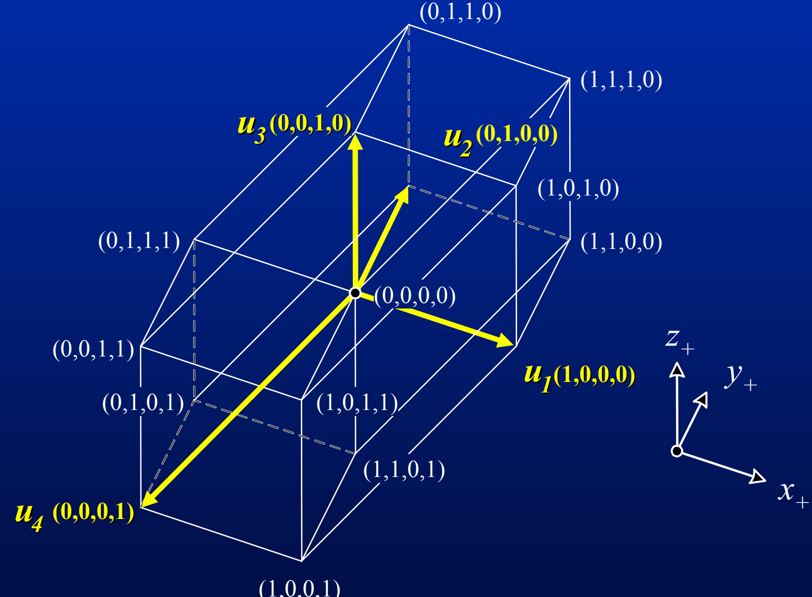
Our Goal

- To utilize 5th degree box spline solid (a projection of 8D hypercube to 3D through a major diagonal (0,…,0) – (1,…,1))
- Direction sets are to be overlapped ($N_{2,2,2,2}$).

$$
\begin{aligned}
\pi_d((1,0,0,0)) &= (1,0,0) = \boldsymbol{u}_1,\\
\pi_d((0,1,0,0)) &= (0,1,0) = \boldsymbol{u}_2,\\
\pi_d((0,0,1,0)) &= (0,0,1) = \boldsymbol{u}_3,\\
\pi_d((0,0,0,1)) &= (-1,-1,-1) = \boldsymbol{u}_4.
\end{aligned}
$$

3D Regular Mesh

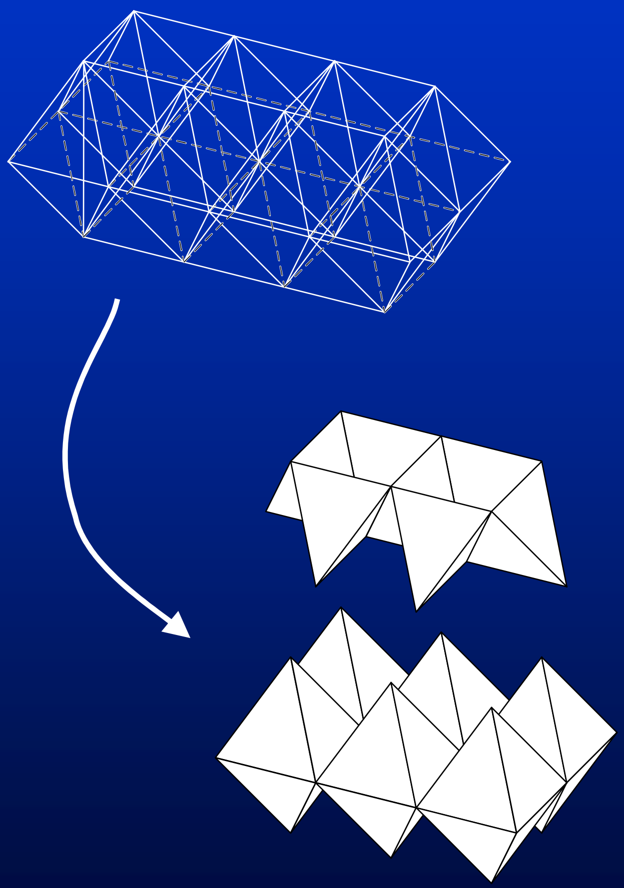
- Projection of direction sets (2 for each u_i)
- It does not form a tessellation of 3D space.



3. Trivariate Box Spline Solids (2)

3D Regular Mesh *(Cont'd)*

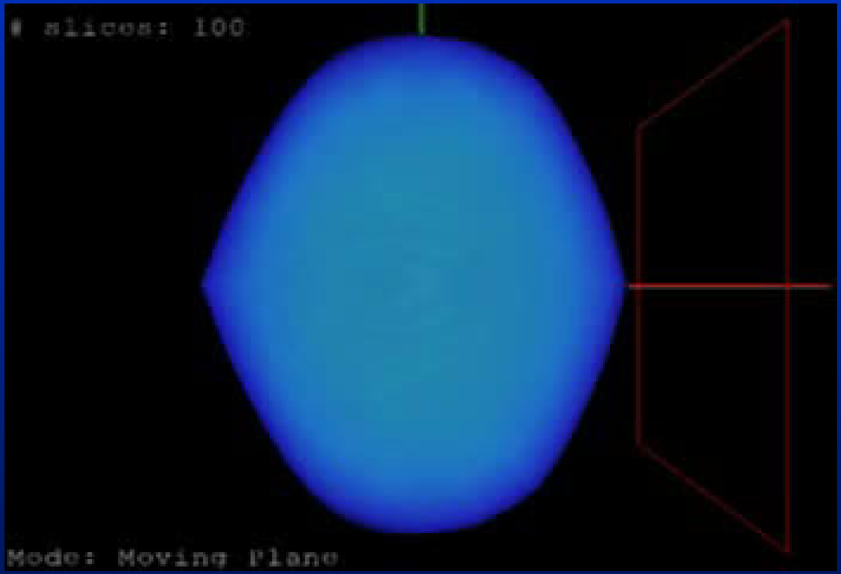
- If the spline is used for meshing purpose (i.e. filtering, FFD), it does not matter.
- However, for the modeling purpose, we have to figure out simplest tessellation based on the projection.
- Octet-truss is a good candidate, which comprises of octahedra with tetrahedra in between.
- Regular valance for each vertex / edges



3. Trivariate Box Spline Solids [3]

Basis Functions over Octet-truss

Basis (Gray Scale)

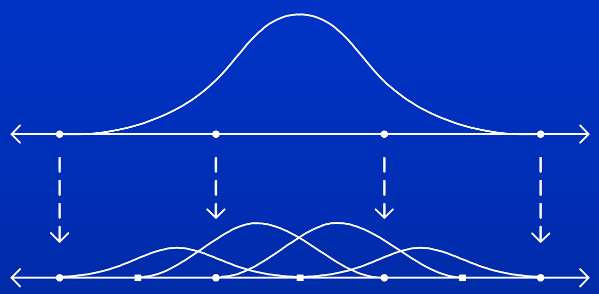


Basis (Spectral)

3. Trivariate Box Spline Solids (4)

Subdivision of Box Splines

- One box spline can be defined by an affine combination of box splines of smaller (half) support (consider the natural half cutting of cubes and their projected images).



$$N_i = \frac{1}{2} \sum_{\hat{j}} \alpha_{i,\hat{j}} \, \hat{N}_{\hat{j}},$$

$$\alpha_{i,\hat{j}} = \begin{cases} 2 & \text{if } i = \hat{j} \\ 1 & \text{if } i \text{ is adjacent to } \hat{j} \\ 0 & \text{otherwise} \end{cases}$$

$$N_i = \frac{1}{2} \sum_{\hat{j}} \alpha_{i,\hat{j}} \, \frac{1}{c} \sum_{\hat{k}} \alpha_{\hat{j},\hat{k}} \, \hat{N}_{\hat{k}}.$$

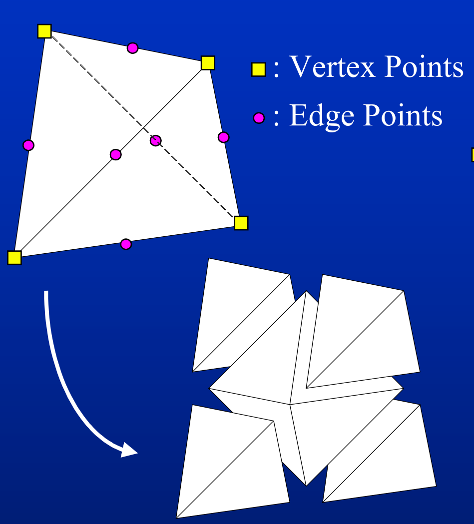
$$S = \sum_{j} w_{\hat{j}} \, \hat{N}_{\hat{j}},$$

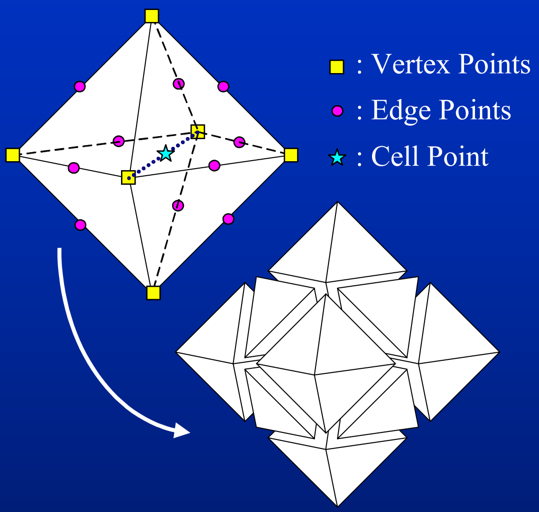
$$w_{\hat{j}} = \frac{1}{2c} \sum_{i} \sum_{\hat{k}} \alpha_{i,\hat{j}} \, \alpha_{\hat{j},\hat{k}} \, v_i = \frac{1}{2c} \sum_{i} \sum_{\hat{k}} \beta_{i,\hat{j},\hat{k}} \, v_i.$$

3. Trivariate Box Spline Solids (5)

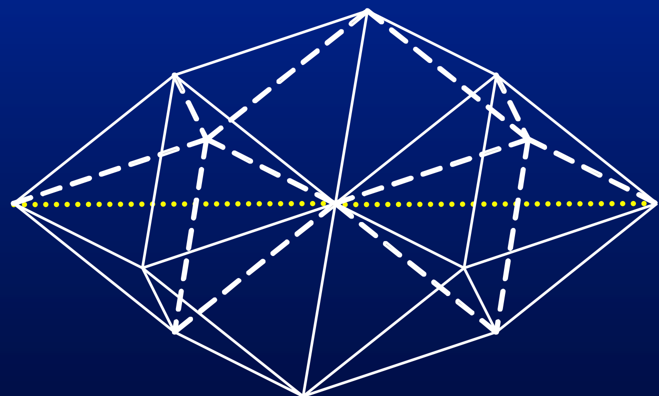
Splitting the Domain

- Edge bisection
- Introducing cell points for octahedra



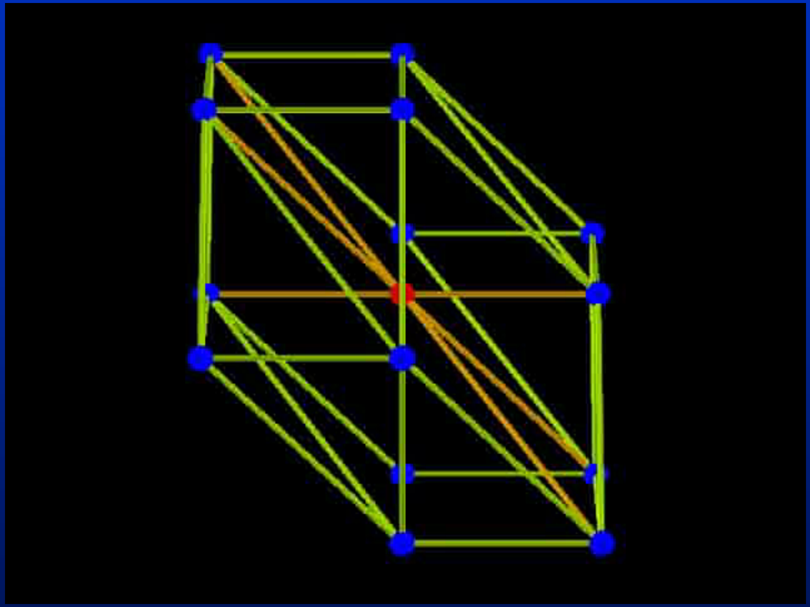


Masks

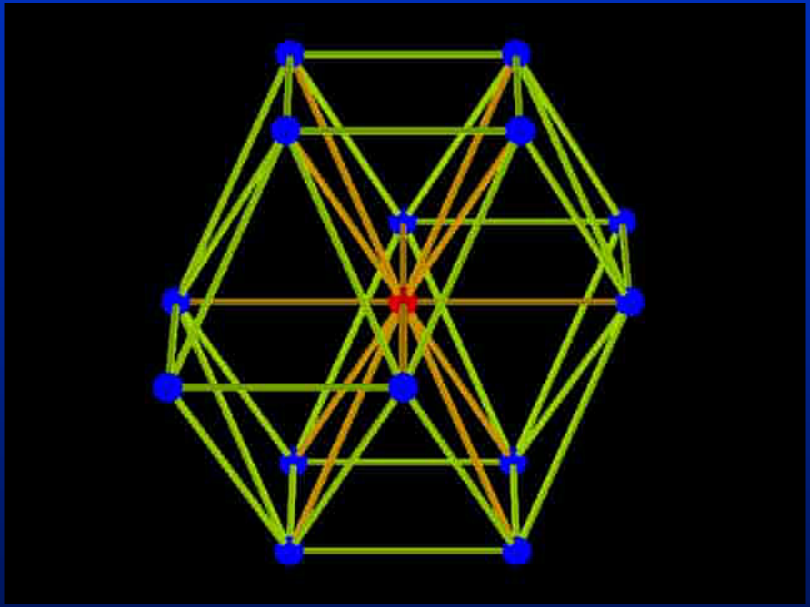


3. Trivariate Box Spline Solids [6]

Masks over Regular Meshes



Mask over a Hypercube



Mask over Octet-truss

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4. Solid Subdivision Rules [1]

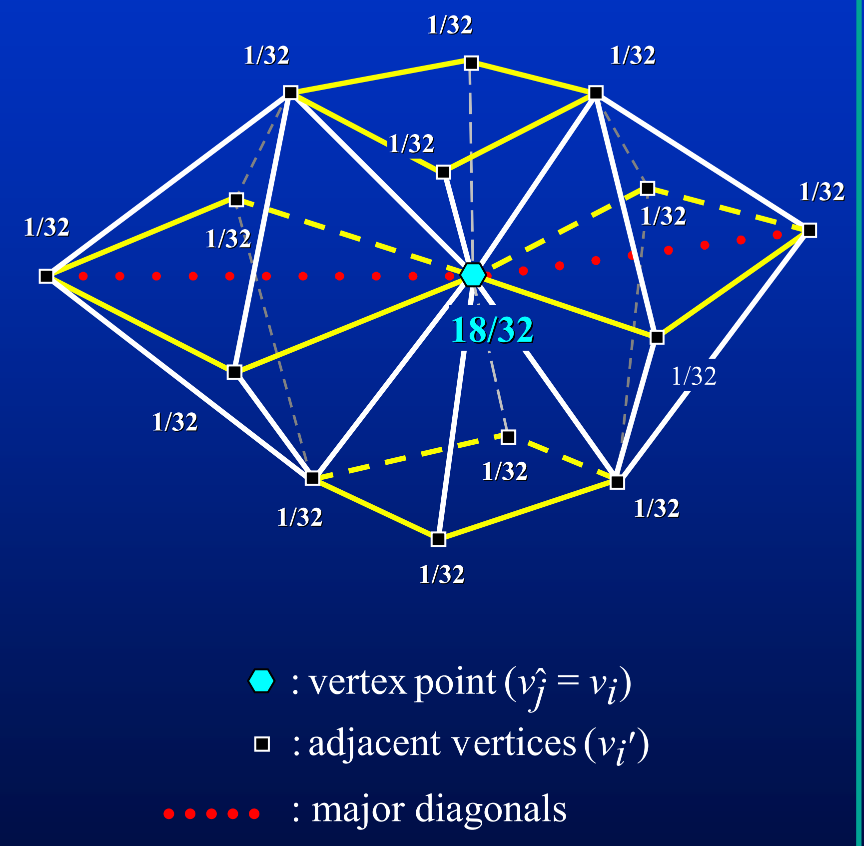
■ *Vertex Points*

$$v_{\hat{j}} = \frac{18}{32} v_i + \frac{1}{32} \sum_{i'} v_{i'},$$

or,

$$v_{\hat{j}} = \frac{18}{32} v_i + \frac{14}{32k} \sum_{i'} v_{i'},$$

for valence k.



4. Solid Subdivision Rules (2)

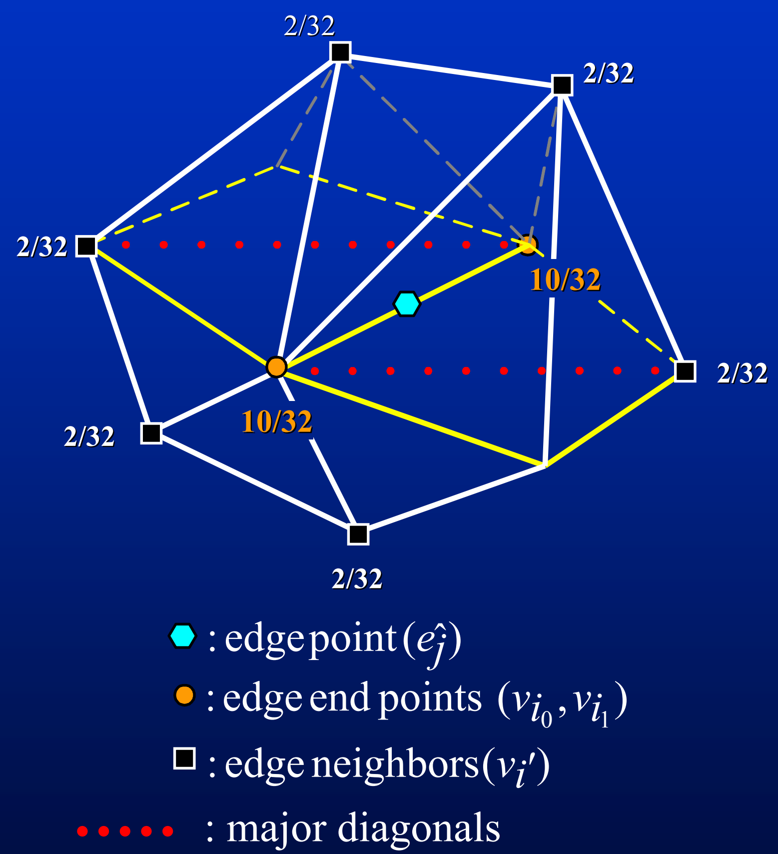
- ***Edge Points***

$$e_{\hat{j}} = \frac{10}{32}\left(v_{i_0} + v_{i_1}\right) + \frac{2}{32}\sum_{i'} v_{i'},$$

or,

$$e_{\hat{j}} = \frac{10}{32}\left(v_{i_0} + v_{i_1}\right) + \frac{12}{32k}\sum_{i'} v_{i'},$$

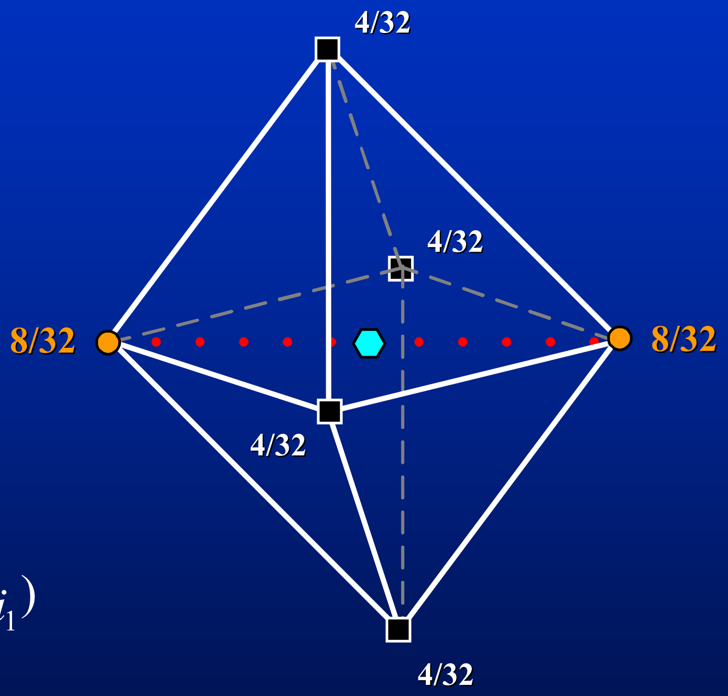
where k: # of edge neighbors.



4. Solid Subdivision Rules (3)

Cell Points

$$c_{\hat{j}} = \frac{8}{32}\left(v_{i_0} + v_{i_1}\right) + \frac{4}{32}\sum_{i'} v_{i'},$$



- : cell point ($c_{\hat{j}}$)
- : major diagonal vertices (v_{i_0}, v_{i_1})
- : cell vertices ($v_{i'}$)
- : major diagonals

4. Solid Subdivision Rules (4)

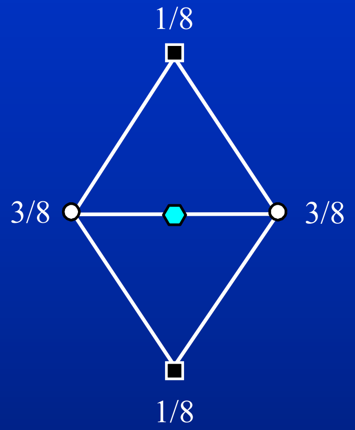
Extraordinary Cases

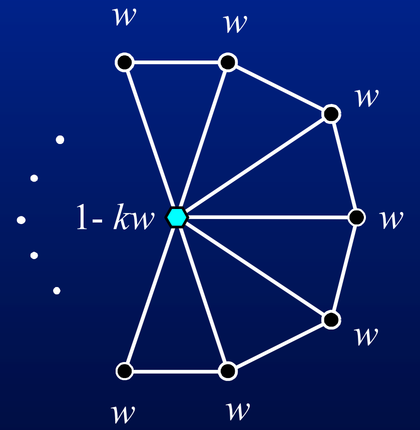
- Vertices with valence $k \neq 14$.
- Edges with # of edge neighbors $\neq 6$.
- Good News: The irregularity will not be propagated over the levels (only contained in control points / edges).
- Bad News: It is very hard to analyze and acquire exact weight for continuity (esp. because it involves not only extraordinary vertices but extraordinary edges).
- Our suggested value at least keep convergence.
- Further study is required (maybe a new analyzing tool for 3D like eigenvalue / characteristic map method in surface case?).

4. Solid Subdivision Rules (5)

Boundary Surface

- We use a modified Loop's scheme (Warren '99) for our b-rep.
- Since Loop's surface is a box spline surface, it would be good choice.
- Or, one can use partial projection of 8D hypercube to achieve surfaces (many case analysis required).
- Or, one can use physical notion to drag interface (between boundary and solid inside) a little bit toward the boundaries.





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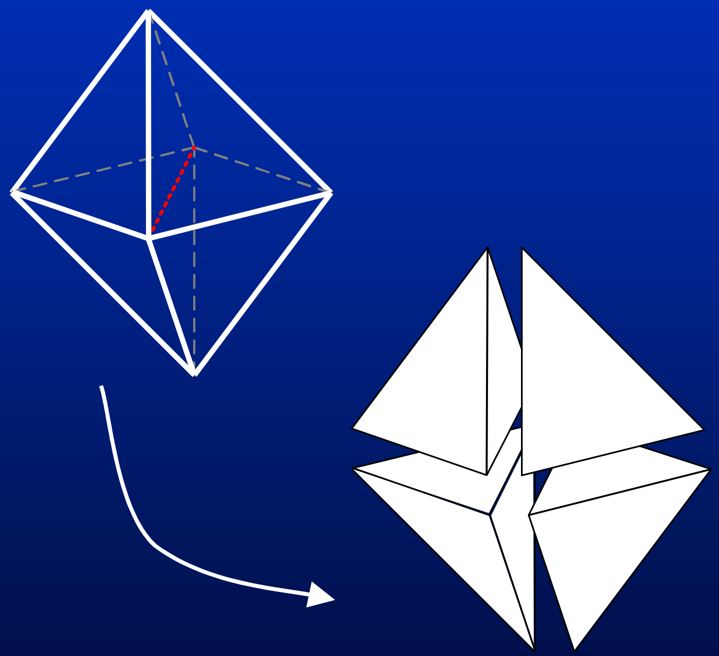
5. Applications (1.1)

Free-Form Deformation

- Procedures
  - *Generate an appropriate mesh that contains the model to be deformed.*
  - *Subdivide the mesh up to the desired level.*
  - *Calculate barycentric coordinates.*
  - *Interactively deform the mesh.*
  - *Recalculate the coordinates.*
- Barycentric coordinates for an octahedron will be obtained after splitting it into 4 sub-tetrahedra.

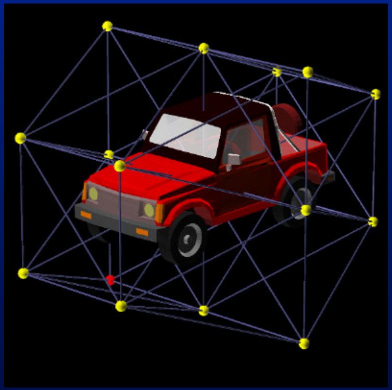
$$p = v_0 + c_1\, u_1 + c_2\, u_2 + c_3\, u_3,$$



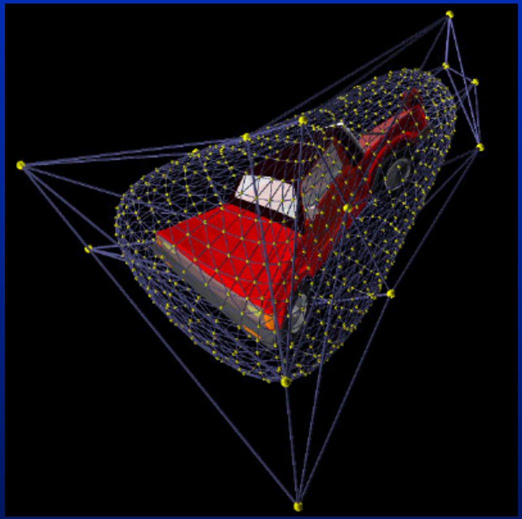
5. Applications (1.2)

Free-Form Deformation Example

Original Model



Solid Mesh

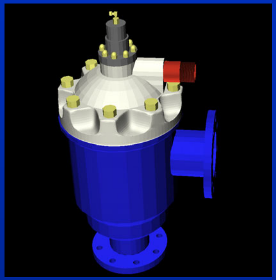


Deformed (Level 2)

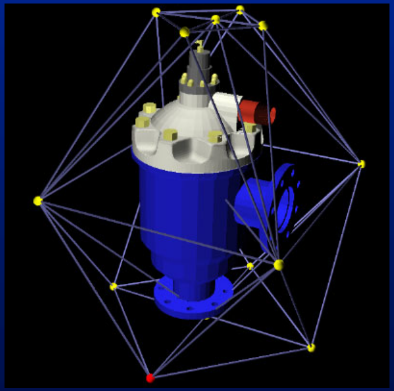
Result

5. Applications (1.3)

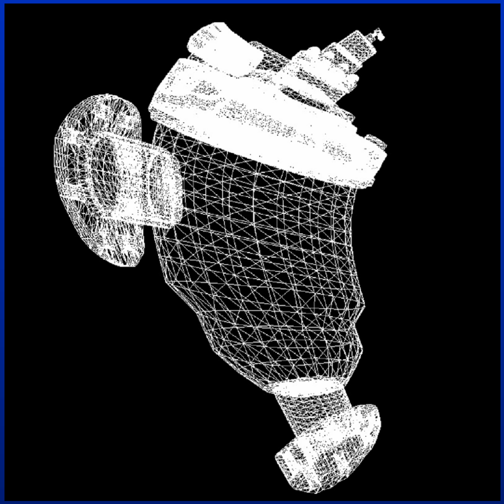
Free-Form Deformation Example (Complex >> 49000 faces)



Original Model



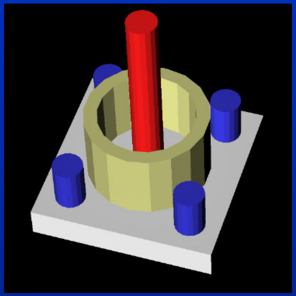
Solid Mesh



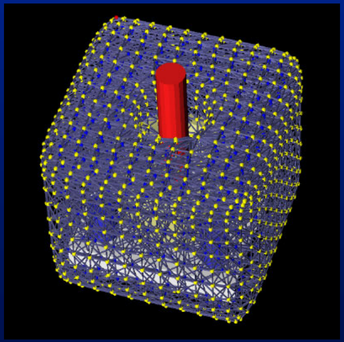
Deformed (Results in both surface rendered and wireframe)

5. Applications (1.4)

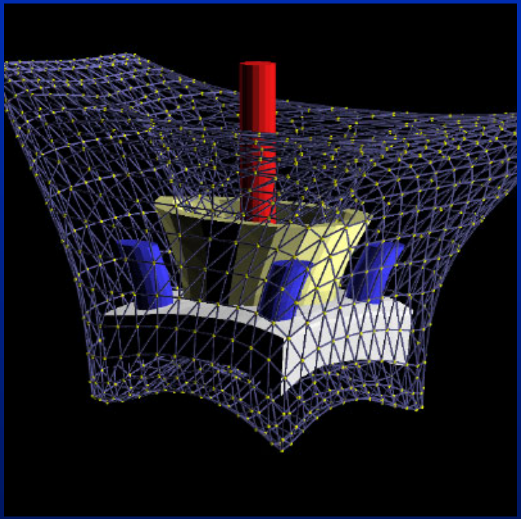
Free-Form Deformation Example (Non-trivial topology)



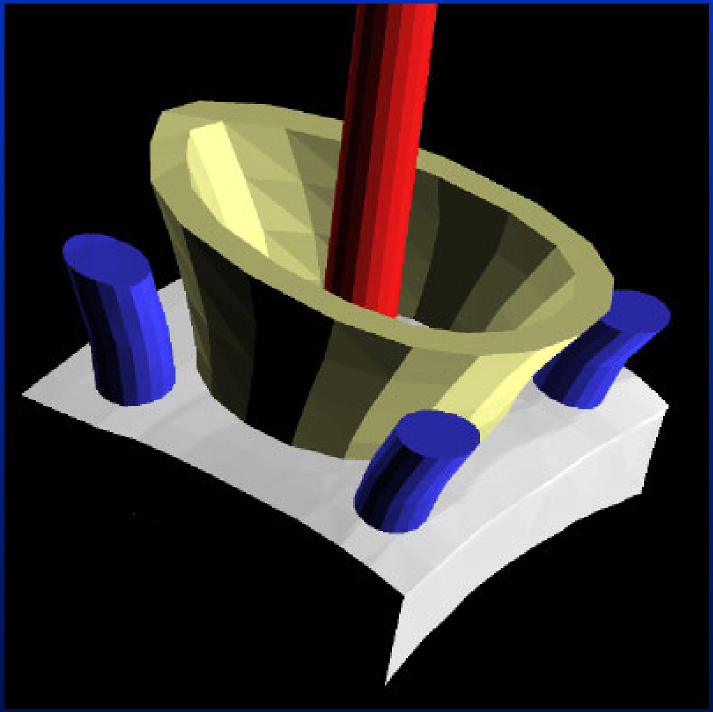
Original Model



Solid Mesh with a hole



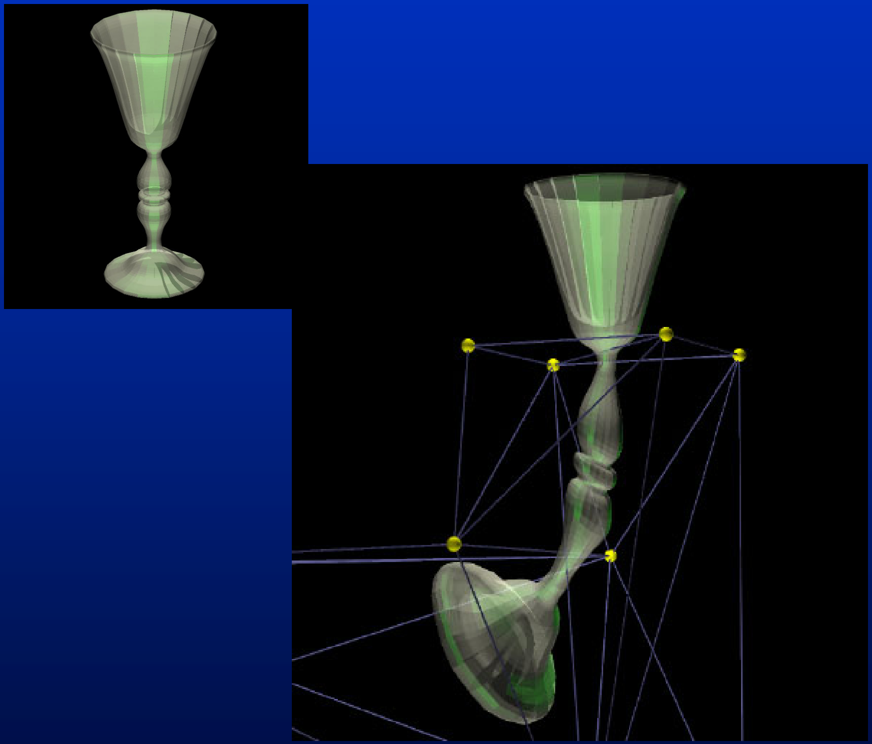
Deformed (Level 2)

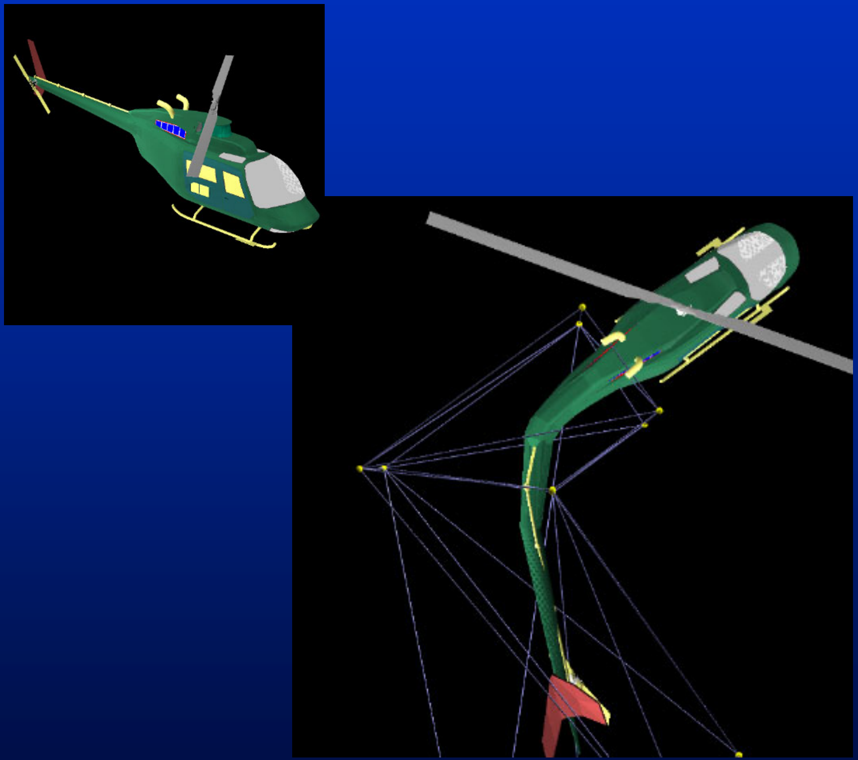


Result (no change in central cylinder)

5. Applications (1.5)

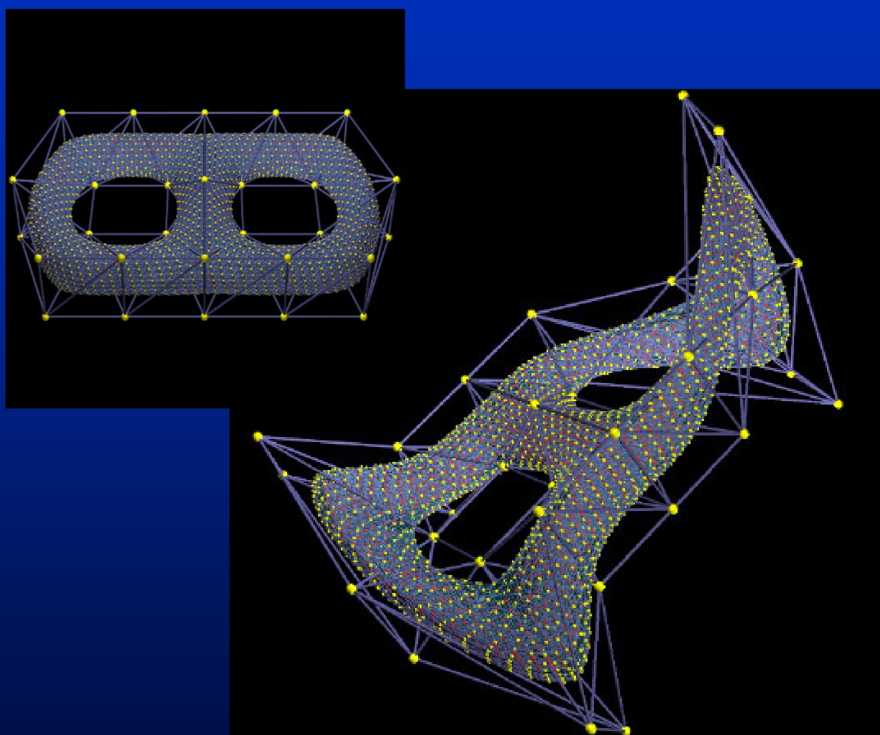
Free-Form Deformation Example (Localized)

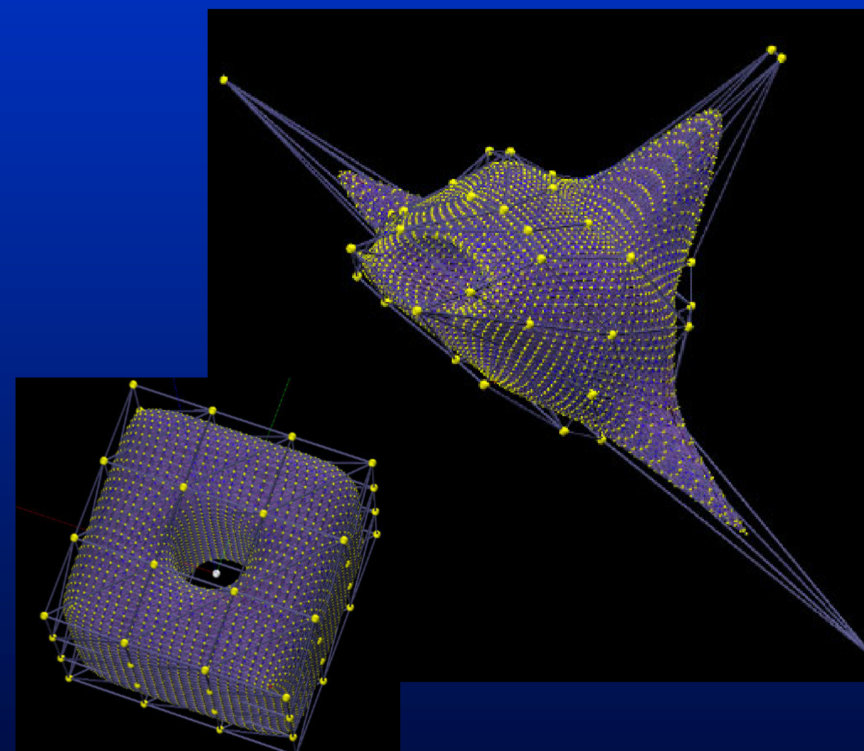




5. Applications (2.1)

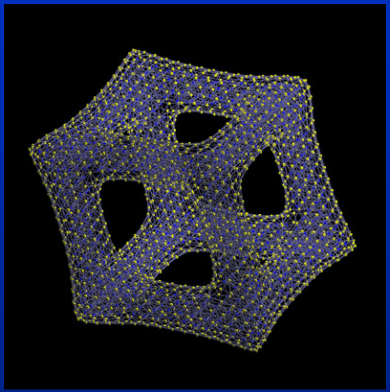
Direct Modeling / Manipulation

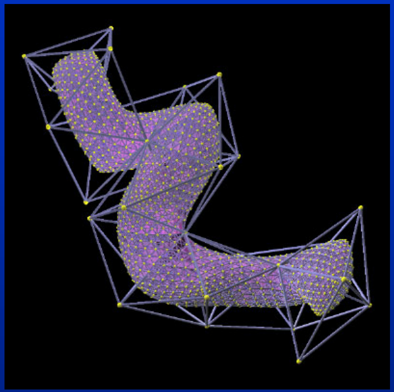


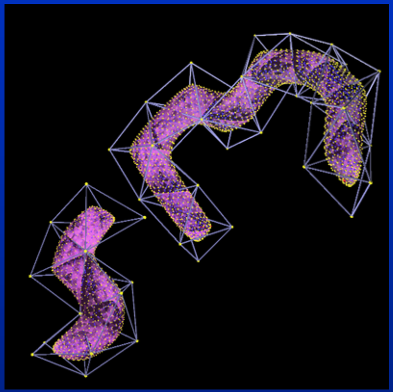


5. Applications (2.2)

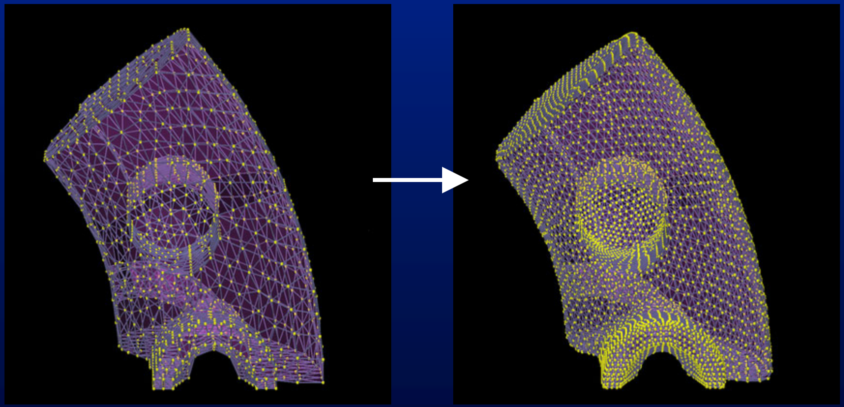
Arbitrary Shapes / Topology





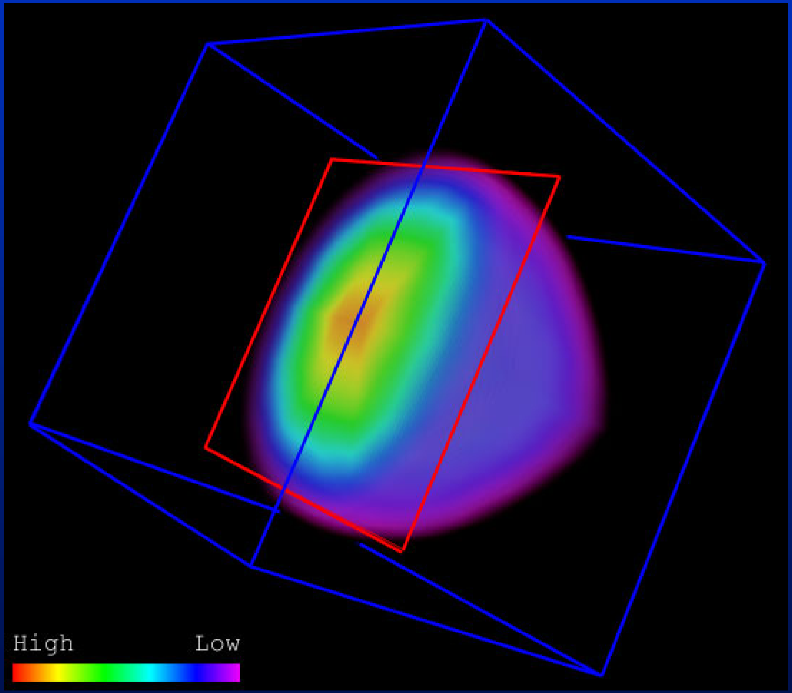


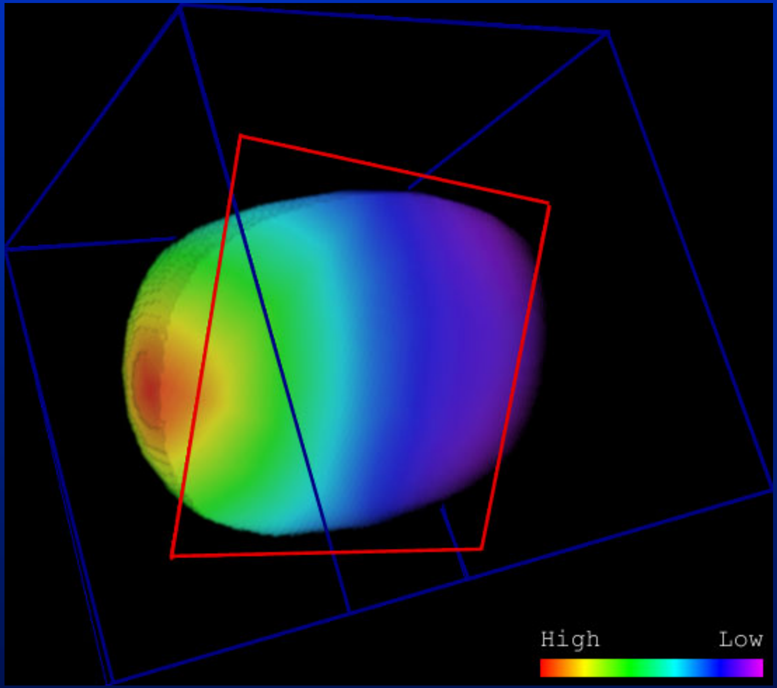
Remeshing Existing Solids for FEM



5. Applications (3)

Representing Material Properties





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6. Conclusion and Future Works [1]

■ ***We have suggested a novel solid subdivision scheme for modeling purpose.***

- Underlying tetrahedral structure gives us more flexibility of shapes than tensor product.
- 12 DOF rather than 6 DOF of tensor product grid → better physics simulation.
- High order of continuity with low degree of basis functions
- Fast and stable subdivision evaluation of solid itself
- Choice of boundary representations

6. Conclusion and Future Works (2)

Future Works

- No rigorous analysis on extraordinary vertex / edge cases. → future research is a must (ongoing).
- Possible implicit application → using tetrahedral grid instead of hexahedral lattice.
- Boundary care. Localized subdivision (easier than hexahedra).
- Robust modeling tools. Direct manipulation on surface / interior.
- Collision detection between models / Boolean operations.
- Physics-based solids. Various PDE models.
- Data fitting (in 3D using volumetric data). Possible medical application.
- Another basis function! → a simplex spline solid and its (possible) subdivision.