An Interpolatory Subdivision for Volumetric Models over Simplicial Complexes

Yu-Sung Chang

Kevin T. McDonnell

Hong Qin

Department of Computer Science

Stony Brook University

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1. Introduction and Background (1)

Why Subdivision Method?

- An efficient way to represent geometric shapes such as curves and surfaces by successive refinements
- Theoretically mature, yet growing technology

Recent Interest in Subdivision Method

- Need for multiresolution techniques and challenges of large, complex models
- Movie industries, CAGD, commercial rendering tools (Maya, Render Man, etc.)

1. Introduction and Background (2)

Why So Popular?

- In essence, the process can be written as *successive affine combinations*. → **Simple**

$$\mathbf{p}^{k+1} = S\mathbf{p}^{k} = S\begin{pmatrix} \mathbf{p}_0^k \\ \vdots \\ \mathbf{p}_m^k \end{pmatrix}$$

where $\mathbf{p}^k$ is a matrix whose row vectors are control points at level k.

S is a subdivision matrix.

- In general, we get spline surfaces in the limit. → **Practical**
- Fast and stable evaluation → **Robust**

1. Introduction and Background (3)

Approximating Subdivision

- Chaikin ('74) : A curve by chopping corners
- Catmull-Clark ('78), Doo-Sabin ('78), Loop ('87) : Generating spline surfaces in the limit
- $\sqrt{3}$ – (Kobbelt '00), 4-8 subdivision (Velho '01) : Slow cell increase

Interpolatory Subdivision

- 4-point curve scheme (Dyn *et al*. '87)
- Butterfly scheme (Dyn *et al*. '90) : Interpolatory surface
- Kobbelt's quadrilateral interpolatory scheme ('96)

1. Introduction and Background (4)

Subdivision Analysis

- Doo and Sabin's analysis ('78)
- Eigen analysis and characteristic map : Peters and Reif ('95), Schweitzer ('96), Zorin ('97) and many other researchers
- Interpolatory scheme convergence : Dyn *et al*. ('90)
- Mostly surface cases

Volumetric Subdivision

- MacCracken and Joy ('96) : Tensor-product of Catmull-Clark
- Bajaj *et al*. ('02) : MLCA scheme
- Kobbelt, Weimer and Warren ('98, '99) : Variational subdivision

1. Introduction and Background (4)

Motivation

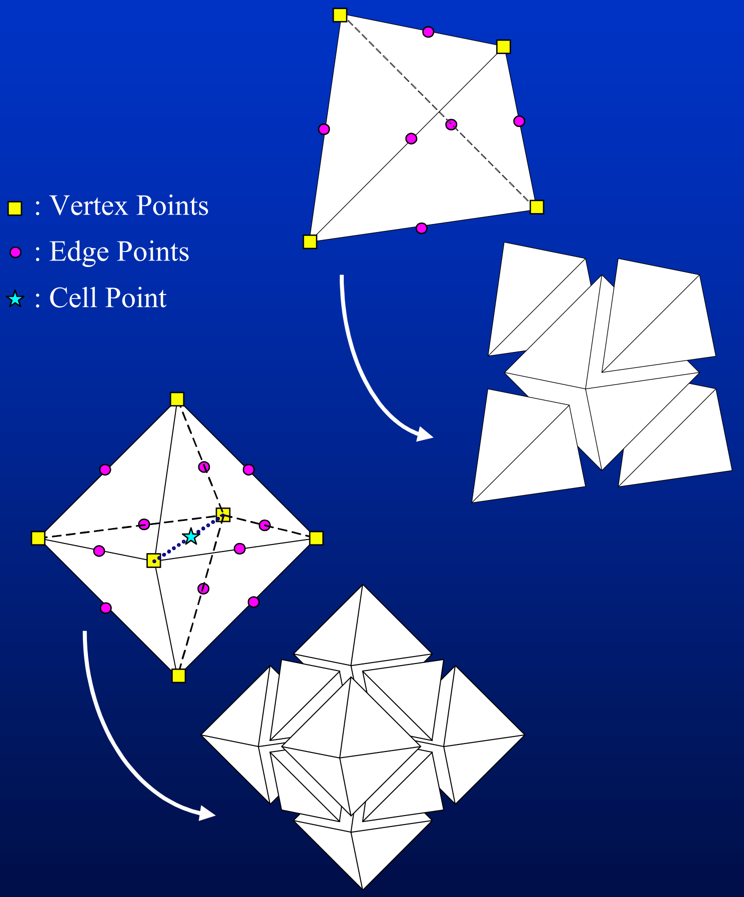
- Volumetric subdivision has been underdeveloped.
- Many attractive properties for solid modeling
  - *Multiresoultion, Numerical stability, Arbitrary topology handling, Simplicity in implementation*
- Benefits of interpolatory subdivision
  - *Easy to enforce constraint, Direct manipulation, Simplified data fitting and physics simulation*
- Non-tensor product subdivision scheme (tetrahedral based)

Contents

2. Subdivision Mesh (1)

Simplicial Complex Mesh in 3D

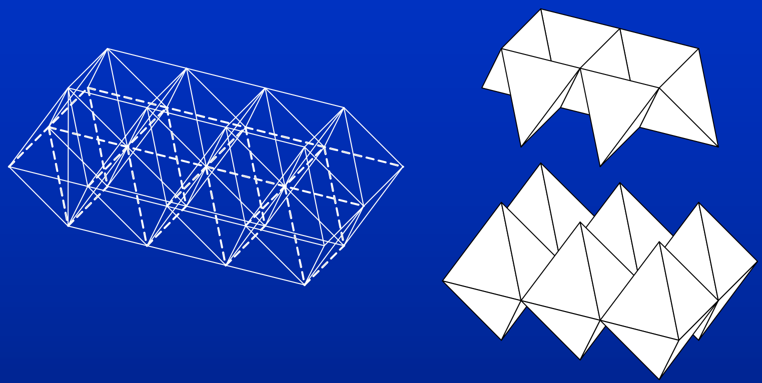
- Tetrahedral mesh
- Simple edge-bisection (face-splitting) of a tetrahedron generates an octahedron inside. → Ambiguity and bookkeeping problems
- We keep the octahedra that occur during the process. → *Octet-truss* structure

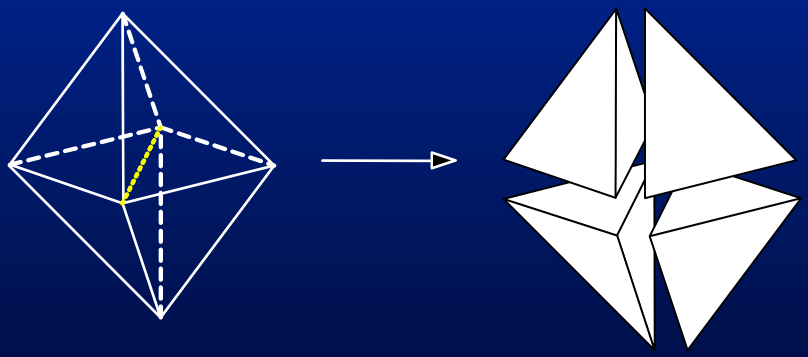


2. Subdivision Mesh (2)

Octet-truss

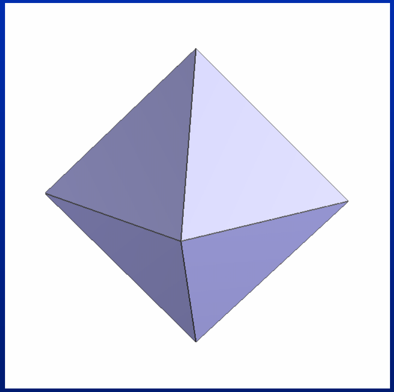
- A mesh structure that consists of tetrahedra and octahedra.
- Each octahedron keeps information of *major diagonal*.
- Octahedra can be split into 4 tetrahedra by the diagonals, if desired. → A conversion into simplicial complex



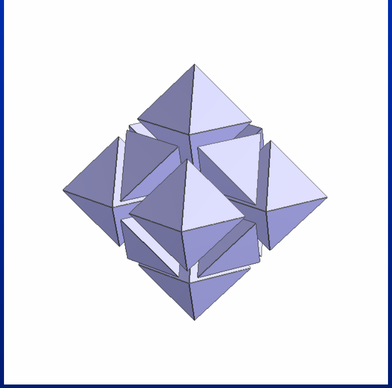


2. Subdivision Mesh (3)

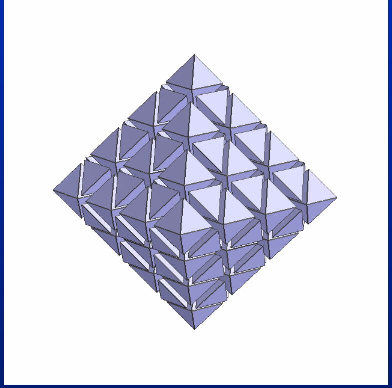
Subdividing the Mesh



Level 1



Level 2



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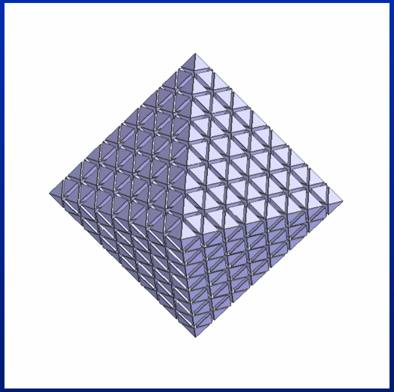

Level 4

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2. ***Subdivision Mesh***
3. ***Subdivision Rules***
4. ***Analysis***
5. ***Examples***
6. ***Conclusion and Future Work***

3. Subdivision Rules (1)

The Process

- A linear interpolation with perturbation

$$\mathbf{p}' = \frac{1}{2}(\mathbf{p}_0 + \mathbf{p}_1) + f(w, \bar{\mathbf{q}}, \bar{\mathbf{r}})$$

w: tension control

$\mathbf{q}, \mathbf{r}$: one and two neighbors

- If w is zero, then the process is a linear interpolation.
- w is chosen in $[0, 1/8)$ to assure the convergence of the subdivision.

3. Subdivision Rules (2)

The Rules (1)

- Vertex point
  - *Interpolating → geometrically invariant*

$$\mathbf{v}^{k+1} = \mathbf{p}^{k}$$

3. Subdivision Rules (3)

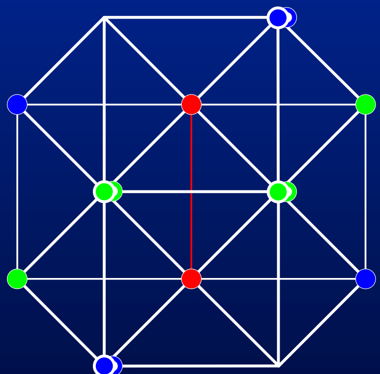
The Rules (2)

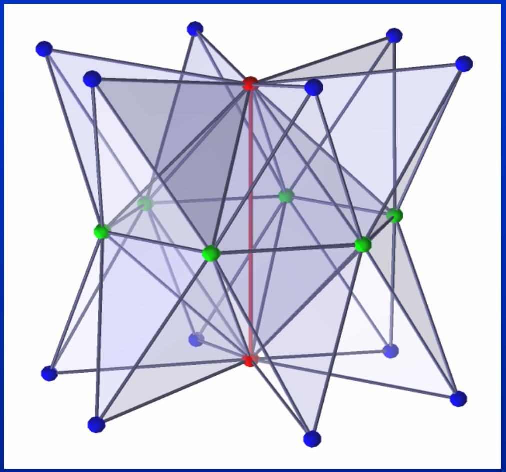
- Edge point:

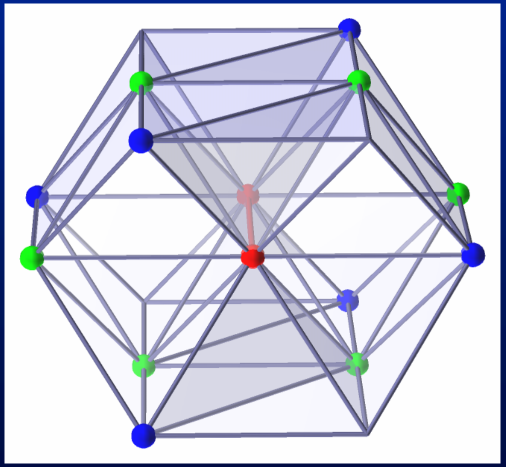
$$\mathbf{e}^{k+1} = \frac{1}{2}(\mathbf{p}_0^k + \mathbf{p}_1^k) + \frac{M}{N} w \sum_{i=0}^{N-1} \mathbf{q}_i^k - w \sum_{j=0}^{M-1} \mathbf{r}_j^k$$

where N, M; Number of one and two neighbors, respectively.

- Weighted average of one and two neighbors





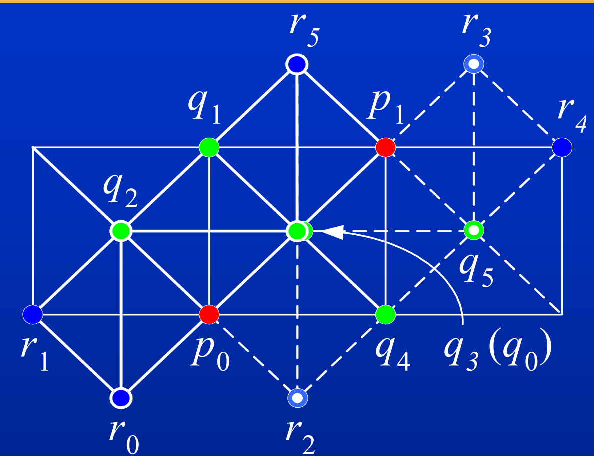


3. Subdivision Rules (4)

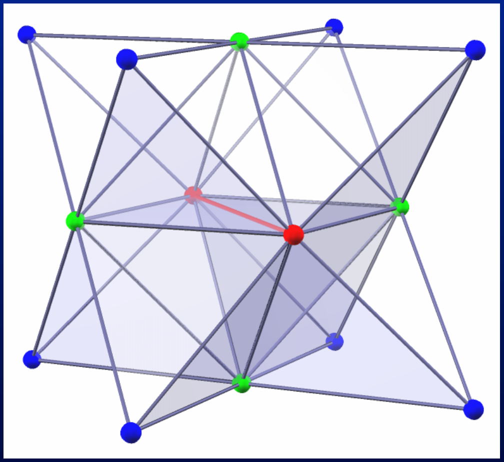
The Rules (3)

- Cell point:

$$\mathbf{c}^{k+1} = \frac{1}{2}(\mathbf{p}_0^k + \mathbf{p}_1^k) + w\sum_{i=0}^{5}\mathbf{q}_i^k - w\sum_{j=0}^{5}\mathbf{r}_j^k$$



  - *An application of the edge rule on the major diagonals*

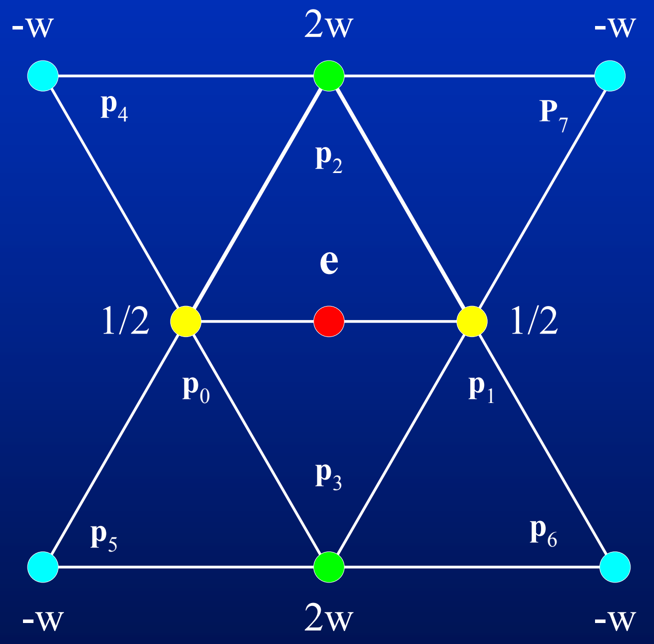


- Modified rule for the simplicity

3. Subdivision Rules (5)

Special Cases

- Boundary representation → Butterfly scheme
- Edges with incomplete neighbors (*i.e.* edges between the boundary and the interior) → Linear interpolation



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4. Analysis (1)

The Continuity on Regular Topology

- Techniques
  - *A relation between a subdivision process and its generating function (Warren and Weimer '01)*
  - *Using the norm and the convergence of the difference subdivision process (Dyn* et al. *'90)*
- The process is convergent when $w < 1/8$.
- The limit solid is C^1 continuous except a finite number of extraordinary topologies when $w \leq 1/16$.
- More details in the paper

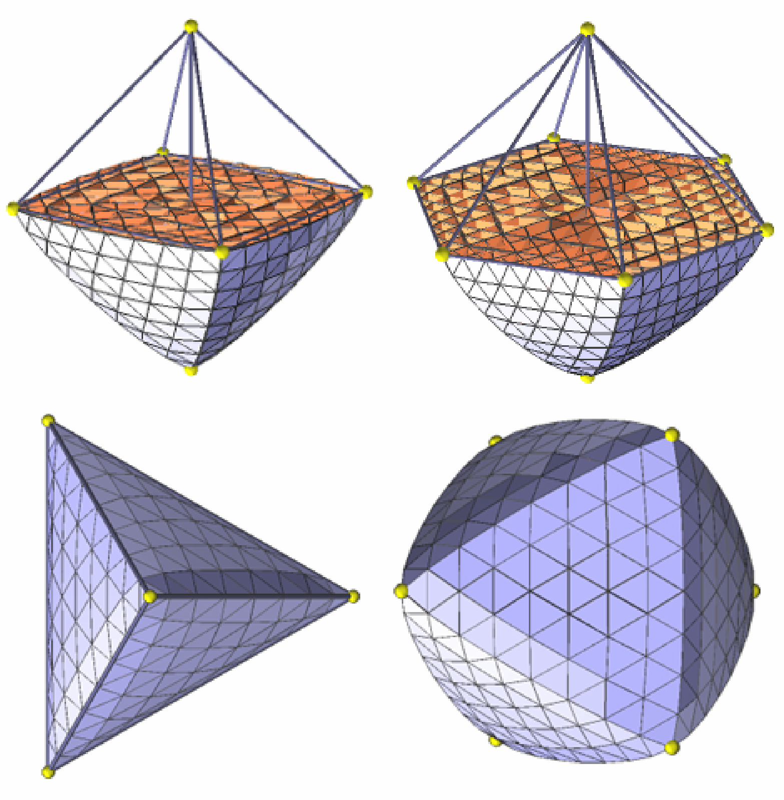
4. Analysis (2)

The Continuity over Irregular Topologies

- Two types of extraordinary cases
  - *Extraordinary edges and extraordinary vertices*
- Eigen analysis of the subdivision matrix and characteristic map

$$1 = \lambda_0 > \lambda_1 \geq \lambda_2 \geq \lambda_3 > \lambda_4 > \cdots > \lambda_n$$

  - *Numerical results suggest that the process is C^1 in most cases.*



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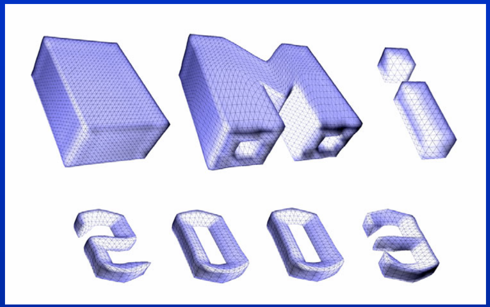
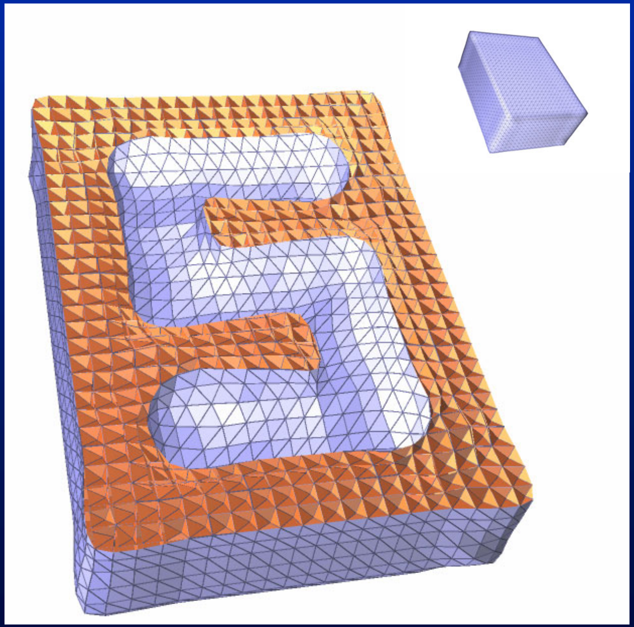
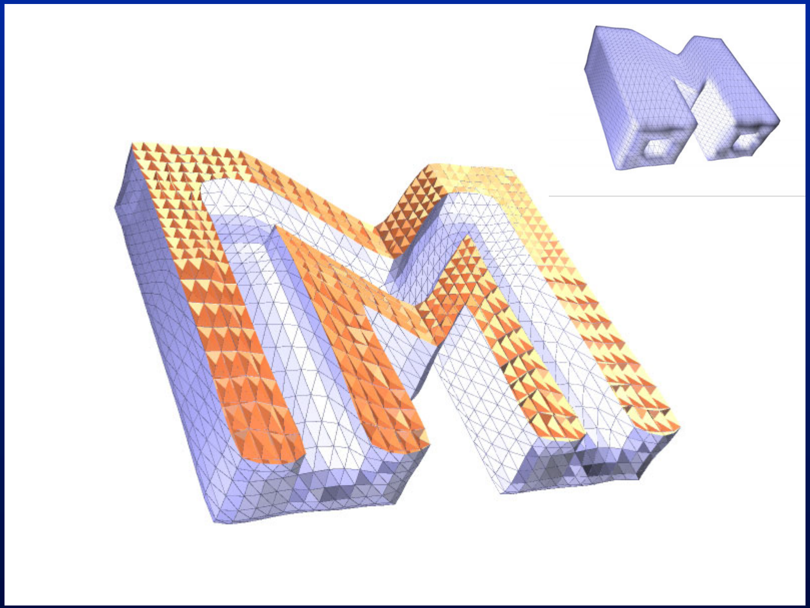
5. Example (1)

Complex Geometry Example



- Our subdivision can easily treat objects with non-trivial topologies.

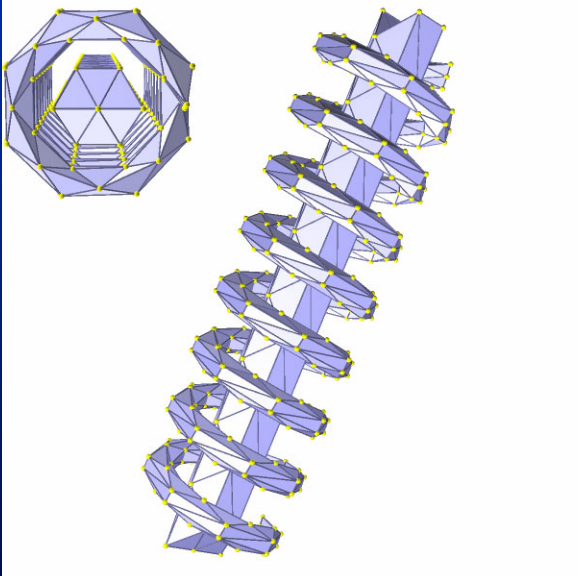


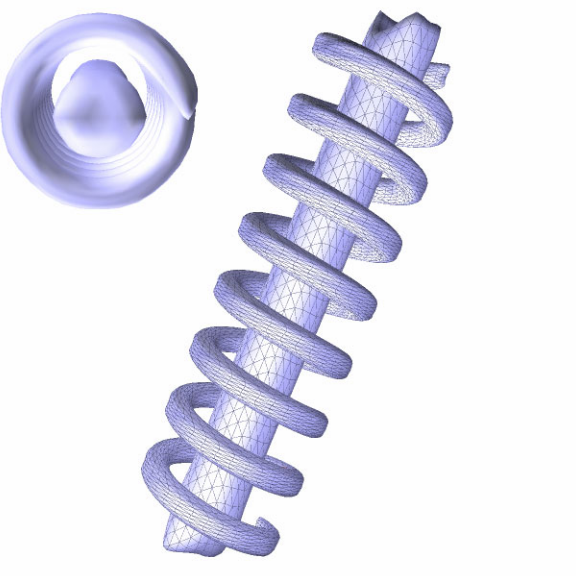


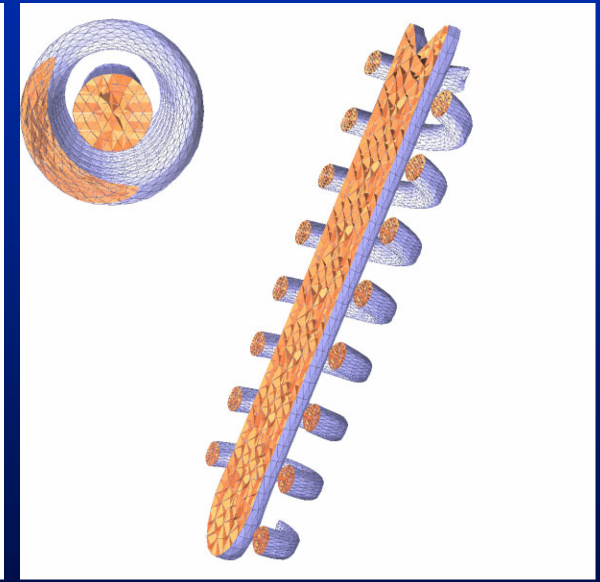
5. Example (2)

Practical Model (Spiral)

- Handling of practical models (e.g. sweeping objects) to meet industrial needs



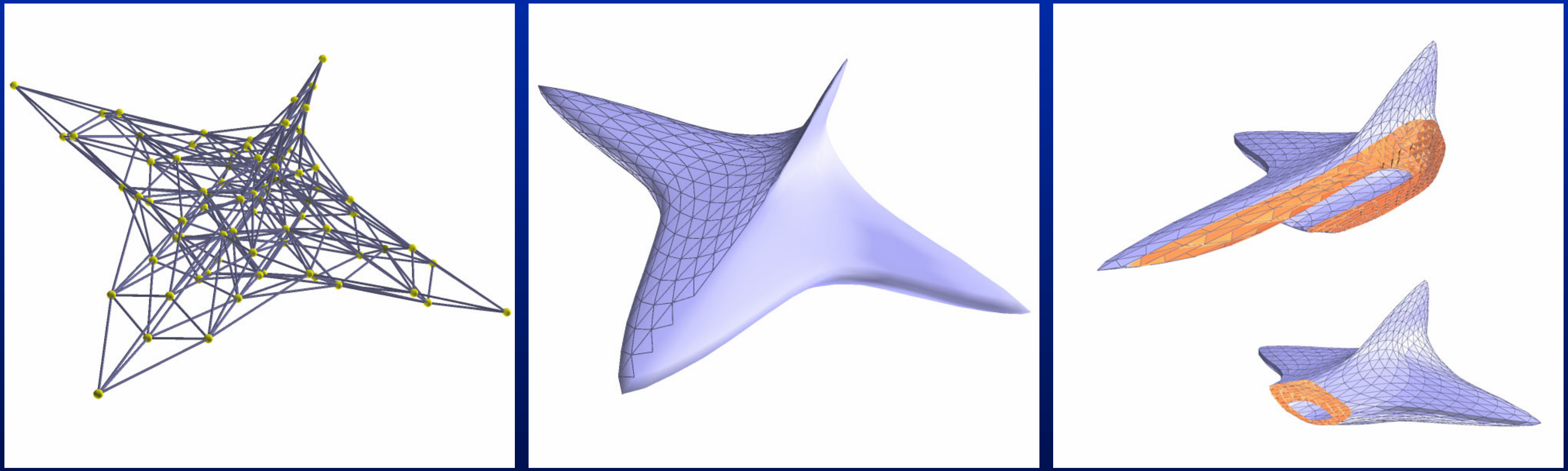




5. Example (3)

Direct Model Design

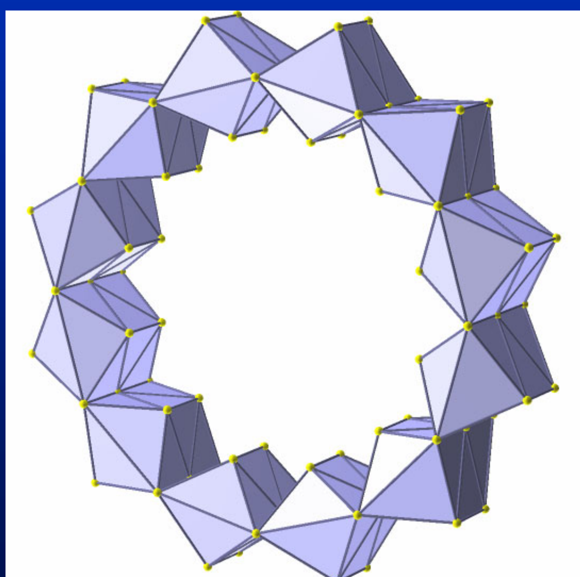
- A user can manipulate control points intuitively for interactive model design.

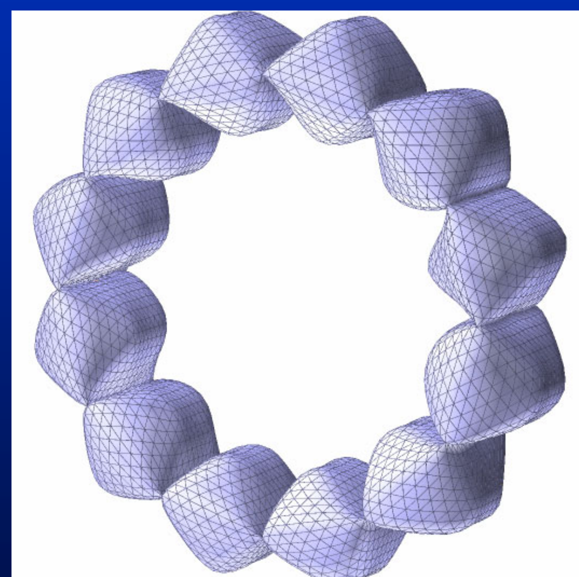


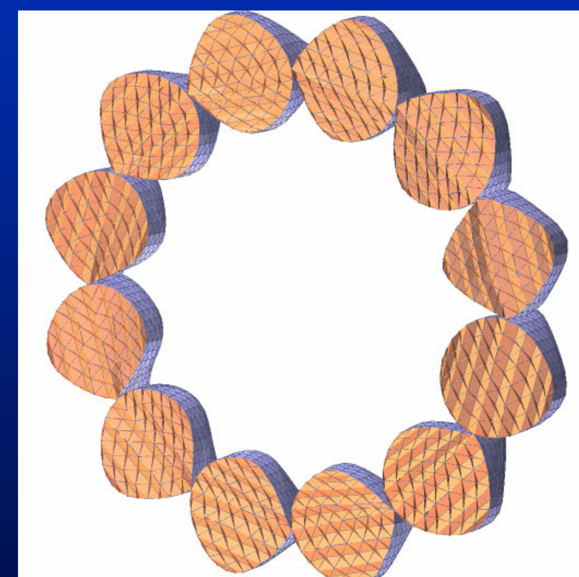
5. Example (4)

Non-manifold Examples (1)

- A degenerating case : A solid combined with 1D segments



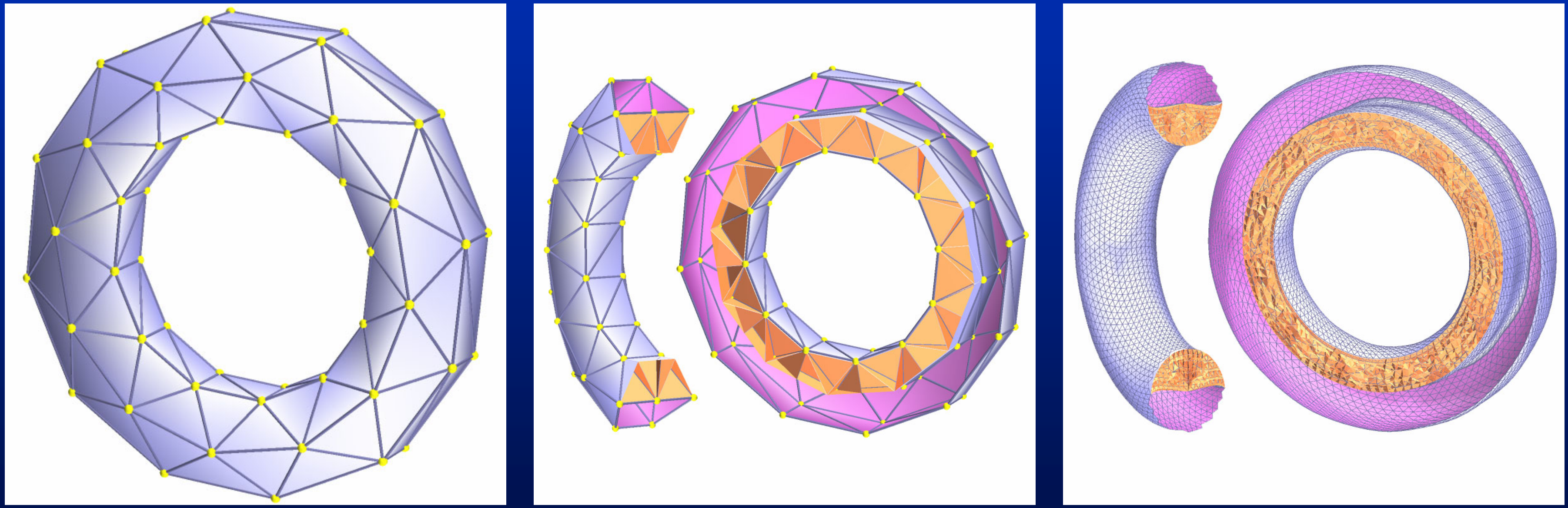




5. Example (5)

Non-manifold Examples

- Solids and surfaces in a single representation

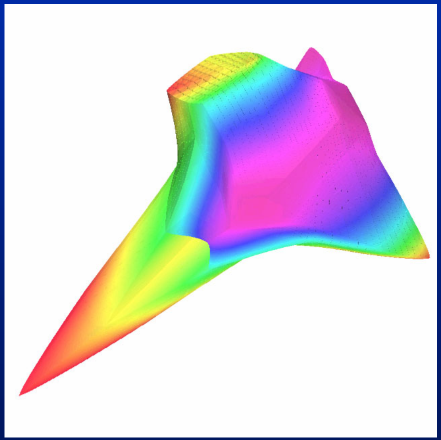


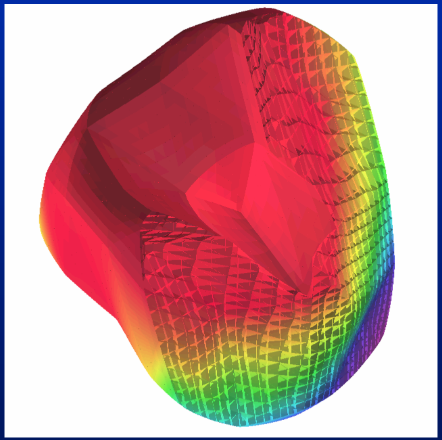
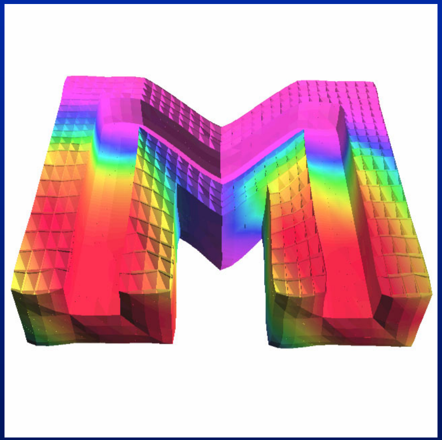
* Pink region indicates surface area.

5. Example (6)

Material Representation (Non-homogeneous)

- Colors represent the material property distribution inside the solids.



* The material property has been diffused by applying heat transfer equation to the initial control points. Finer levels are interpolated by our subdivision.

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6. Conclusion and Future Work [1]

Conclusion

- We suggested a novel interpolatory subdivision for solid objects.
- Underlying structure mesh is octet-truss. → It is *not* tensor product based schemes.
- The conversion between the mesh and a simplicial complex is trivial.
- The limit solid has C^1 continuity almost everywhere.
- The scheme can handle non-trivial topologies with ease.
- Non-manifold objects can be represented through simple modification of data structure.
- Non-homogeneous material property representation

6. Conclusion and Future Work (2)

Future Work

- It still requires more rigorous analysis for extraordinary cases.
  → Not well-established for solid subdivision schemes.
- Future applications
  - *Data fitting with high dimensional scientific data*
  - *Data conversion from existing volumetric data*
  - *Interactive modeling*
  - *Feature modeling*
  - *Local subdivision, multiresolution approach*

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The End